3/9/18

HW: "Inverses of Functions" HW section
Test 2 on Wednesday 3/14

AIM: How do we find Inverses of Functions?

Warm Up:

1) GIVEN $f(x)=2x+9$ *and* $g(x)=\dfrac{x-9}{2}$. EVALUATE $f(g(2))$

Name: ______ [redacted] ______ Date: ______

FUNCTION COMPOSITION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given $f(x) = 3x - 4$ and $g(x) = -2x + 7$ evaluate:

(a) $f(g(0))$

$g(0) = -2(0) + 7 = 0 + 7 = 7$

$f(7) = 3(7) - 4 = 21 - 4 = 17$

(b) $g(f(-2))$

$f(-2) = 3(-2) - 4 = -6 - 4 = -10$

$g(-10) = -2(-10) + 7 = 20 + 7 = 27$

(c) $f(f(3))$

$f(3) = 3(3) - 4 = 9 - 4 = 5$

$f(5) = 3(5) - 4 = 15 - 4 = 11$

(d) $(g \circ f)(6)$

$f(6) = 3(6) - 4 = 18 - 4 = 14$

$g(14) = -2(14) + 7 = -28 + 7 = -21$

(e) $(f \circ g)(5)$

$g(5) = -2(5) + 7 = -10 + 7 = -3$

$f(-3) = 3(-3) - 4 = -9 - 4 = -13$

(f) $(g \circ g)(2)$

$g(2) = -2(2) + 7 = -4 + 7 = 3$

$g(3) = -2(3) + 7 = -6 + 7 = 1$

2. Given $h(x) = x^2 + 11$ and $g(x) = \sqrt{x - 2}$ evaluate:

(a) $h(g(18))$

$g(18) = \sqrt{18 - 2} = \sqrt{16} = 4$

$h(4) = (4)^2 + 11 = 16 + 11 = 27$

(b) $g(h(4))$

$h(4) = (4)^2 + 11 = 16 + 11 = 27$

$g(27) = \sqrt{27 - 2} = \sqrt{25} = 5$

(c) $(g \circ g)(11)$

$g(11) = \sqrt{11 - 2} = \sqrt{9} = 3$

$g(3) = \sqrt{3 - 2} = \sqrt{1} = 1$

(d) $h(h(0))$

$h(0) = (0)^2 + 11 = 0 + 11 = 11$

$h(11) = (11)^2 + 11 = 121 + 11 = 132$

(e) $(h \circ g)(38)$

$g(38) = \sqrt{38 - 2} = \sqrt{36} = 6$

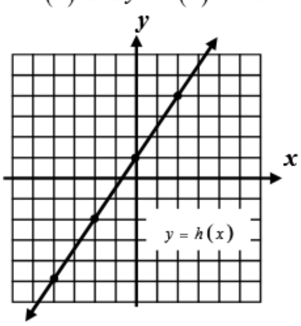
$h(6) = (6)^2 + 11 = 36 + 11 = 47$

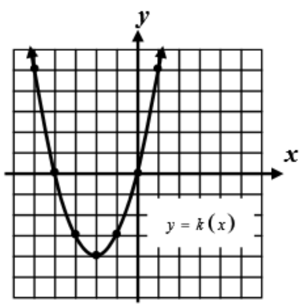
(f) $(g \circ h)(0)$

$h(0) = (0)^2 + 11 = 0 + 11 = 11$

$g(11) = \sqrt{11 - 2} = \sqrt{9} = 3$

3. The graphs of $y = h(x)$ and $y = k(x)$ are shown below. Evaluate the following based on these two graphs.





(a) $h(k(-2))$

$k(-2) = -4$

$h(-4) = -5$

(b) $(k \circ h)(0)$

$h(0) = 1$

$k(1) = 5$

(c) $h(h(-2))$

$h(-2) = -2$

$h(-2) = -2$

(d) $(k \circ k)(-2)$

$k(-2) = -4$

$k(-4) = 0$

4. If $g(x)=3x-5$ and $h(x)=2x-4$ then $(g\circ h)(x)=?$

(1) $6x-17$ (3) $5x-9$

(2) $6x-14$ (4) $x-1$

$g(2x-4)=3(2x-4)-5$
$=6x-12-5=6x-17$

$\boxed{(1)}$

5. If $f(x)=x^2+5$ and $g(x)=x+4$ then $f(g(x))=$

(1) x^2+9 (3) $4x^2+20$

(2) $x^2+8x+21$ (4) x^2+21

$f(x+4)=(x+4)^2+5$
$=(x+4)(x+4)+5$
$=x^2+4x+4x+16+5$
$=x^2+8x+21$

$\boxed{(2)}$

APPLICATIONS

6. Scientists modeled the intensity of the sun, I, as a function of the number of hours *since* 6:00 a.m., h, using the function $I(h)=\frac{12h-h^2}{36}$. They then model the temperature of the soil, T, as a function of the intensity using the function $T(I)=\sqrt{5000I}$. Which of the following is closest to the temperature of the soil at 2:00 p.m. ?

8hrs

(1) 54 (3) 67

(2) 84 (4) 38

$I(8)=\frac{12(8)-(8)^2}{36}=0.\overline{8}$ or $8/9$

$T(8/9)=\sqrt{5000(8/9)}=66.\overline{66}\approx 67$

$\boxed{(3)}$

7. Physics students are studying the effect of the temperature, T, on the speed of sound, S. They find that the speed of sound in meters per second is a function of the temperature in degrees Kelvin, K, by $S(K)=\sqrt{410K}$. The degrees Kelvin is a function of the temperature in Celsius given by $K(C)=C+273.15$. Find the speed of sound when the temperature is 30 degrees Celsius. Round to the nearest *tenth*.

$K(30)=30+273.15=303.15$
$S(303.15)=\sqrt{410(303.15)}=352.5499...$
≈ 352.5 **meters per second**

REASONING

8. Consider the functions $f(x)=2x+9$ and $g(x)=\frac{x-9}{2}$. Calculate the following.

(a) $g(f(15))$

$f(15)=2(15)+9=39$
$g(39)=\frac{39-9}{2}=\frac{30}{2}=15$

(b) $g(f(-3))$

$f(-3)=2(-3)+9=-6+9=3$
$g(3)=\frac{3-9}{2}=\frac{-6}{2}=-3$

(c) $g(f(x))$

$g(2x+9)=\frac{2x+9-9}{2}$
$=\frac{\cancel{2}x}{\cancel{2}}=x$

(d) What appears to always be true when you compose these two functions?

It appears that when we compose these two functions they always give us back the original input.

***Exercise* #1:** Consider the two linear functions given by the formulas $f(x)=\frac{3x+7}{2}$ and $g(x)=\frac{2x-7}{3}$.

(a) Calculate $f(5)$ and $g(11)$.

$f(5)=\frac{3(5)+7}{2}=11$

$g(11)=\frac{2(11)-7}{3}=5$

(b) Calculate $f(0)$ and $g\left(\frac{7}{2}\right)$.

$f(0)=\frac{3(0)+7}{2}=\frac{7}{2}$

$g\left(\frac{7}{2}\right)=\frac{2\left(\frac{7}{2}\right)-7}{3}=\frac{0}{3}=0$

(c) Calculate $f(g(-1))$.

$g(-1)=\frac{2(-1)-7}{3}=-3$

$f(-3)=\frac{3(-3)+7}{2}=\boxed{-1}$

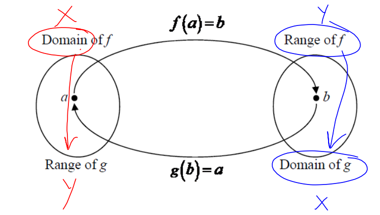
(d) Without calculation, determine the value of $f(g(\pi))$.

π

b/c f and g are inverses.

The two functions seen in Exercise #1 are inverses because they literally "undo" one another. The general idea of inverses, $f(x)$ and $g(x)$, is shown below in the mapping diagram.

Domain of "f" becomes Range of "g"



Range of "f" becomes Domain of "g"

Exercise #2: If the point $(-3, 5)$ lies on the graph of $y = f(x)$, then which of the following points must lie on the graph of its inverse?

switch them

(1) $(3, -5)$ $\quad$ (3) $\boxed{(5, -3)}$

(2) $(-5, 3)$ $\quad$ (4) $\left(-\frac{1}{3}, \frac{1}{5}\right)$

$(-3, 5) \longrightarrow (5, -3)$

Inverse functions have their own special notation. It is shown in the box below.

INVERSE FUNCTION NOTATION
If a function $y = f(x)$ has an inverse that is also a function we represent it as $y = f^{-1}(x)$.

Exercise #3: The linear function $f(x) = \frac{2}{3}x - 2$ is shown graphed below. Use its graph to answer the following questions.

(a) Evaluate $f^{-1}(2)$ and $f^{-1}(-4)$.

y-value of

where is $y=-4$ on $f(x)$

$\boxed{f^{-1}(2) = 6}$ $\quad$ $\boxed{f^{-1}(-4) = -3}$

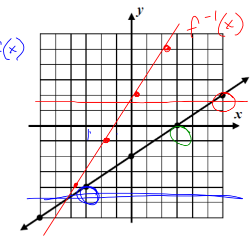
$f(6) = 2$ $\quad$ $f(-3) = -4$

(b) Determine the y-intercept of $f^{-1}(x)$.

(x-intercept on f(x))

y-intercept of $f^{-1}(x) = 3$

(c) On the same set of axes, draw a graph of $y = f^{-1}(x)$.



Points on f(x)	switch x with y	$f^{-1}(x)$
$(-3, -4)$	$\longrightarrow$	$(-4, -3)$
$(6, 2)$		$(2, 6)$
$(3, 0)$		$(0, 3)$
$(0, -2)$		$(-2, 0)$

✱ To find the equation of an inverse: Switch x and y

$f(x) = \frac{2}{3}x - 2$

OR

$y = \frac{2}{3}x - 2$

Inverse

$x = \frac{2}{3}y - 2$

solving for y

$x = \frac{2}{3}y - 2$

$+2 \qquad +2$

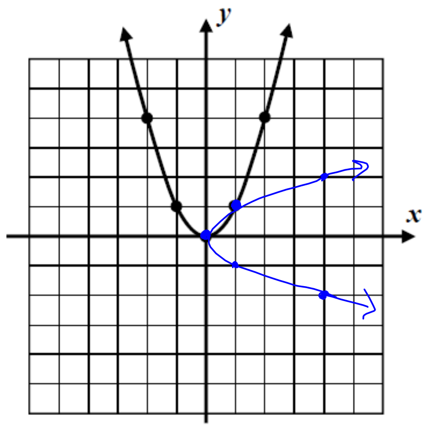
$\left(\frac{3}{2}\right) x + 2 = \cancel{\frac{2}{3}} y$

$\frac{3}{2}x + 3 = y$

$f^{-1}(x) = \frac{3}{2}x + 3$

***Exercise* #4:** A table of values for the simple quadratic function $f(x) = x^2$ is given below along with its graph.

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4



(a) Graph the inverse by switching the ordered pairs.

x	4	1	0	1	4
$f^{-1}(x)$	-2	-1	0	1	2

(b) What do you notice about the graph of this function's inverse?

Inverse is NOT a function.

EXISTENCE OF INVERSE FUNCTIONS

A function will have an inverse that is also a function if and only if it is one-to-one. Hence, a quick way to know if a function has an inverse that is also a function is to apply the Horizontal Line Test.

If a function is 1 to 1 then its inverse is a function.